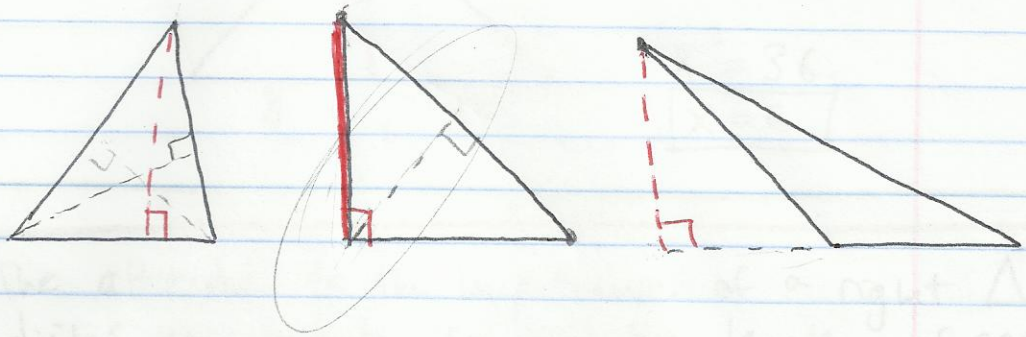
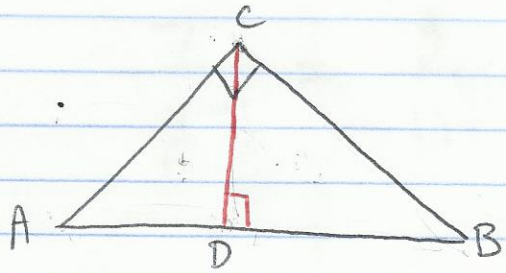
7-4 Similarity in Right Δs

Altitude: the perpendicular segment from a vertex of a triangle to the line containing the opposite side.



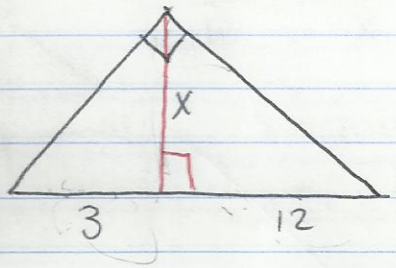
Geometric Mean: the geometric mean of a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$ or $x = \sqrt{ab}$

The altitude to the hypotenuse of a right Δ divides it into 2 Δs that are similar to the original Δ and similar to each other.



$\Delta ABC \sim \Delta ACD \sim \Delta CBD$

The length of the altitude to the hypotenuse of a right $\triangle$ is the geometric mean of the hypotenuse segments.

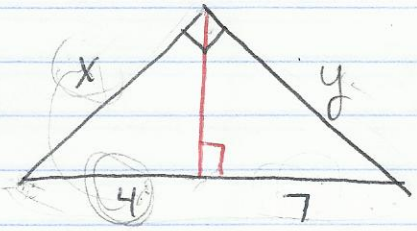


$$\frac{3}{x} = \frac{x}{12}$$

$$x^2 = 36$$

$$\boxed{x = 6}$$

The altitude to the hypotenuse of a right $\triangle$ divides the triangle so that the length of each leg is the geometric mean of the length of the adjacent hypotenuse segment and the length of the hypotenuse.



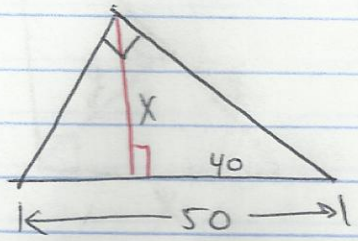
$$\frac{x}{4} = \frac{11}{x}$$

$$x^2 = 44$$

$$x = \sqrt{44} = \boxed{2\sqrt{11}}$$

$$\frac{7}{y} = \frac{y}{11}$$

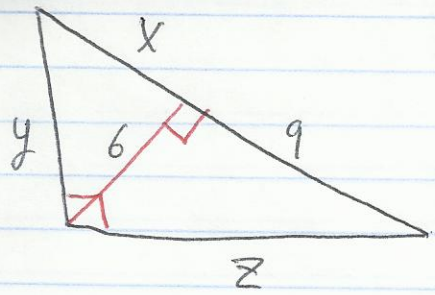
Practice:

① 

$$\frac{x}{10} = \frac{40}{x}$$

$$x^2 = 400$$

$$\boxed{x = 20}$$

② 

$$\frac{6}{x} = \frac{9}{6} \qquad \frac{y}{4} = \frac{13}{y}$$

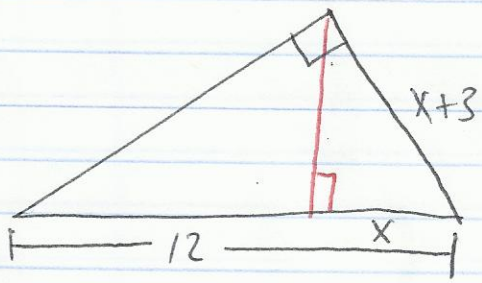
$$9x = 36 \qquad y^2 = 52$$

$$\boxed{x = 4} \qquad y = \sqrt{52} = \boxed{2\sqrt{13}}$$

$$\frac{z}{9} = \frac{13}{z}$$

$$z^2 = 117$$

$$z = \sqrt{117} = \boxed{3\sqrt{13}}$$

③ 

$$\frac{x+3}{x} = \frac{12}{x+3}$$

$$12x = x^2 + 3x + 3x + 9$$

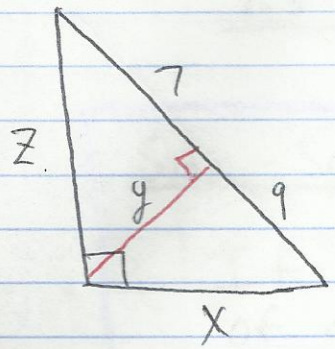
$$12x = x^2 + 6x + 9$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)(x-3) = 0$$

$$\boxed{x = 3}$$

(4)



$\frac{x}{9} = \frac{16}{x}$ $\quad \frac{y}{7} = \frac{9}{y}$

$x^2 = 144$ $\quad y^2 = 63$

$\boxed{x = 12}$ $\quad y = \sqrt{63} = \boxed{3\sqrt{7}}$ (y)

$\frac{z}{7} = \frac{16}{z}$

$z^2 = 112$

$z = \sqrt{112} = \boxed{4\sqrt{7}}$ (z)