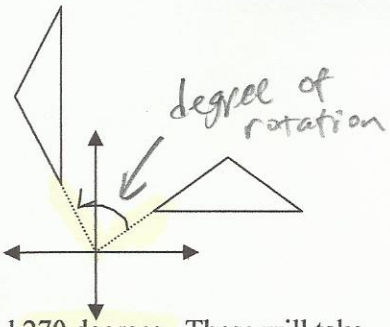
ROTATIONS



A rotation is the spinning of a figure or point around one central point. Rotations are described by the amount of degrees the figure is spun. To figure that amount, measure the angle created by an original point, the center of the rotation, and the image point. The measure of that angle is the degree of the rotation.

In the coordinate plane we will only consider rotations of 90 degrees, 180 degrees, and 270 degrees. These will take our original figure from quadrant I into quadrants II, III and IV, respectively. We will assume that our rotations spin counterclockwise, always using the origin as the central point. This point never changes location—it is called a fixed point.

VOCAB:

"pre-image" → original figure

image → resulting figure of a transformation.

isometry – a transformation in which image and preimage are ≅.

Example 1 - The center of rotation is the origin.

A(8, 2) → A' (-2, 8)

B(3, 5) → B'(-5, 3)

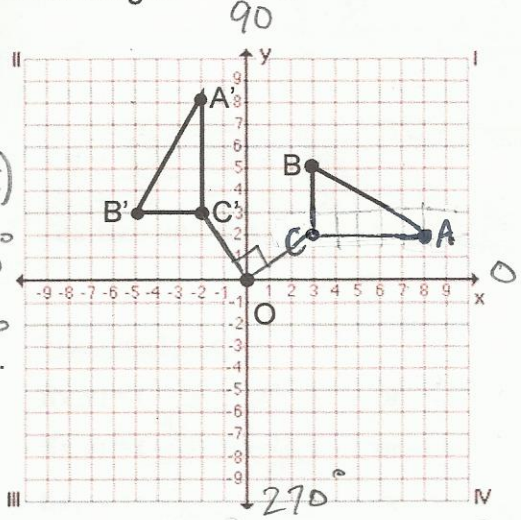
C(3, 2) → C'(-2, 3)

(x, y) ↓ (-y, x)

OC = $\sqrt{13}$; OC'= $\sqrt{13}$

m∠COC' = 90

The degree of this rotation is 90°.



Find the slope of each segment.

$\overline{AB}$ = -3/5 $\overline{A'B'}$ = 5/3

$\overline{AC}$ = 0 $\overline{A'C'}$ = Undefined

$\overline{CB}$ = Und. $\overline{C'B'}$ = 0

Image segment is ⊥ to pre-image segment.

~~m∠ABC = ___~~ ~~m∠A'B'C' = ___~~ $m\angle ABC = m\angle A'B'C'$

AB = $\sqrt{34}$ A'B' = $\sqrt{34}$

BC = 3 B'C' = 3

Are size and shape preserved? Yes

Example 2 - The center of rotation is the origin.

A(8, 2) → A' (-8, -2)

B(3, 5) → B'(-3, -5)

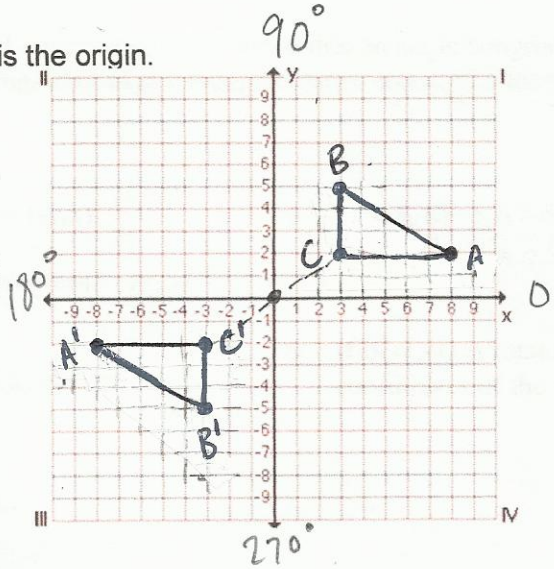
C(3, 2) → C'(-3, -2)

(x, y) ↓ (-x, -y)

OC = $\sqrt{13}$; OC'= $\sqrt{13}$

m∠COC' = 180°

The degree of this rotation is 180°.



Find the slope of each segment.

$\overline{AB}$ = -3/5 $\overline{A'B'}$ = -3/5

$\overline{AC}$ = 0 $\overline{A'C'}$ = 0

$\overline{CB}$ = U. $\overline{C'B'}$ = U.

Image segment is ∥ to pre-image segment.

m∠ABC = ___ m∠A'B'C' = ___

AB = $\sqrt{34}$ A'B' = $\sqrt{34}$

BC = 3 B'C' = 3

Are size and shape preserved? YES

Example 3 - The center of rotation is the origin.

A(8,2) → A' (2,-8)

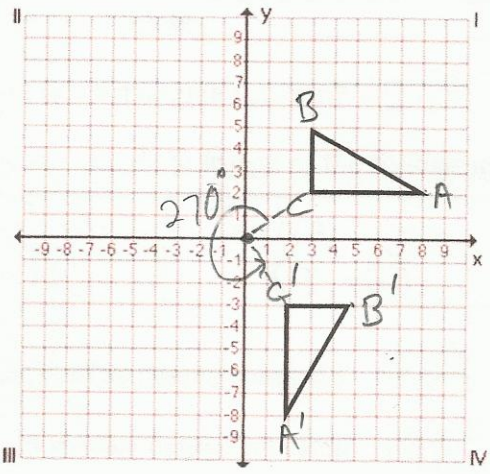
B(3,5) → B'(5,-3)

C(3,2) →C'(2,-3)

OC = $\sqrt{13}$; OC'= $\sqrt{13}$

m∠COC' = 270°

The degree of this rotation is 270°.



Find the slope of each segment.

$\overline{AB}$ = -3/5 $\overline{A'B'}$ = 5/3

$\overline{AC}$ = 0 $\overline{A'C'}$ = u

$\overline{CB}$ = u $\overline{C'B'}$ = 0

Image segment is 1 to pre-image segment.

m∠ABC = ____ m∠A'B'C'= ____

AB = $\sqrt{34}$ A'B' = $\sqrt{34}$

BC = 3 B'C' = 3

Are size and shape preserved? YES

	Functional notation		Equations	
A **90° rotation** is described by:	f(x,y) = (-y, x)	or	x' = -y	y' = x
A **180° rotation** is described by:	f(x,y) = (-x, -y)	or	x' = -x	y' = -y
A **270° rotation** is described by:	f(x,y) = (y, -x)	or	x' = y	y' = -x

Under a rotation:

1. **Size is preserved** - every segment is mapped into a segment congruent to the original segment.
2. **Shape is preserved** - every angle is mapped into an angle congruent to the original angle.
3. **Image figure is congruent to pre-image figure—a.k.a. "Isometry"**

In-Class Exercises

A transformation is described by f(x,y) = (-y,x).

1. Is this a rotation of 90, 180, or 270 degrees? 90°
2. Under this rotation, the image of (5,2) is (-2,5).
3. The image of (-4,3) is (-3,-4)
4. The preimage of (6,-2) is (-2,-6).
5. If A(3,-6) → A'(-6,-3), what is the degree of rotation? 270°
6. If B(-1,-4) is rotated 180°, then what are the coordinates of the image B'? (1,4)