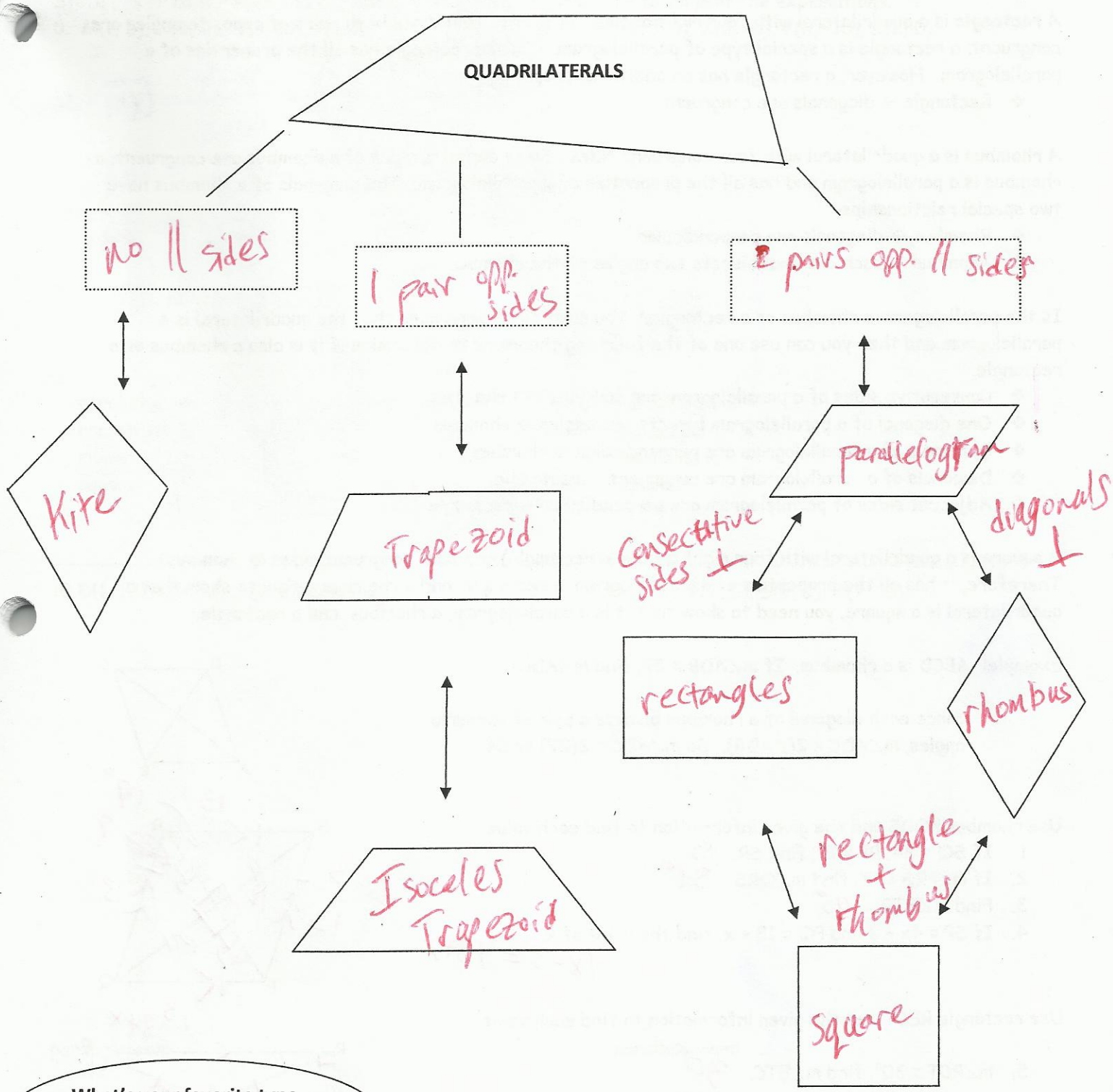
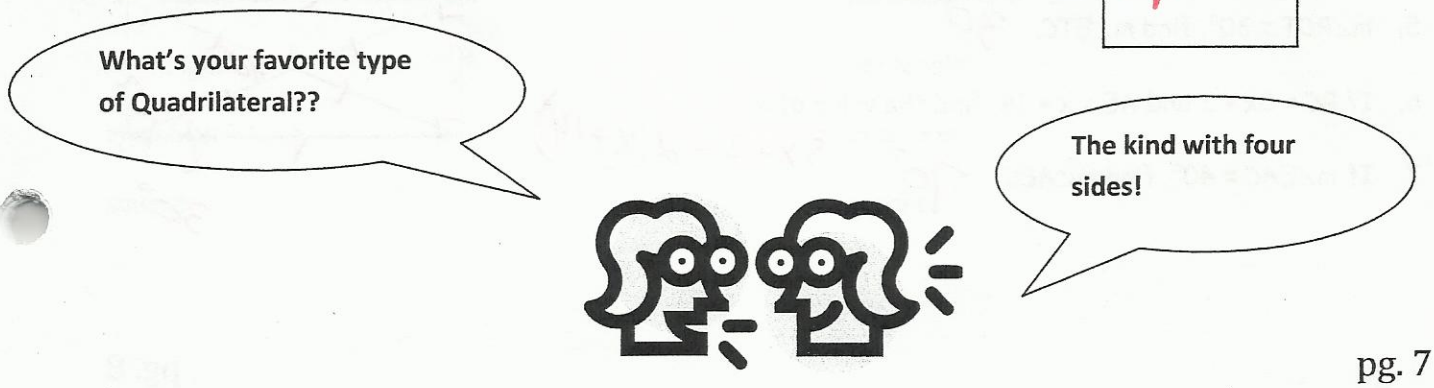
Classifying Quadrilaterals





6-4 Study Guide - Special Parallelograms: Rectangles, Squares and Rhombi

A **rectangle** is a quadrilateral with four right angles. It follows that since both pairs of opposite angles are congruent; a rectangle is a special type of parallelogram. Thus, a rectangle has all the properties of a parallelogram. However, a rectangle has an additional property:

- Rectangle → diagonals are congruent

A **rhombus** is a quadrilateral with four congruent sides. Since opposite sides of a rhombus are congruent, a rhombus is a parallelogram and has all the properties of a parallelogram. The diagonals of a rhombus have two special relationships.

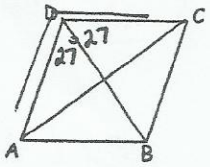
- Rhombus → diagonals are perpendicular
- Rhombus → each diagonal bisects two angles of the rhombus

Is the parallelogram a rhombus or a rectangle? You must first determine that the quadrilateral is a parallelogram and then you can use one of the following theorems to determine if it is also a rhombus or a rectangle.

- ✓ Consecutive sides of a parallelogram are congruent → rhombus
- One diagonal of a parallelogram bisects two angles → rhombus
- Diagonals of a parallelogram are perpendicular → rhombus
- Diagonals of a parallelogram are congruent → rectangle
- ✓ Adjacent sides of parallelogram are perpendicular → rectangle

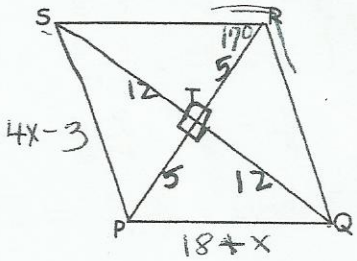
A **square** is a quadrilateral with four right angles (a rectangle) and four congruent sides (a rhombus). Therefore, it has all the properties of a parallelogram, a rectangle, and a rhombus. Also, to show that a quadrilateral is a square, you need to show that it is a parallelogram, a rhombus, and a rectangle.

Example: ABCD is a rhombus. If m∠ADB = 27, find m∠ADC. 54



Since each diagonal of a rhombus bisects a pair of opposite angles, m∠ADC = 2(∠ADB). So m∠ADC = 2(27) or 54.

Use rhombus PQRS and the given information to find each value.

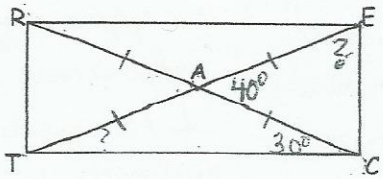


1. If SQ = 24, RP = 10, find SR. 13
2. If m∠PRS = 17, find m∠QRS. 34°
3. Find m∠STR. 90
4. If SP = 4x - 3 and PQ = 18 + x, find the value of x.

   $4x - 3 = 18 + x$ $\quad x = 7$

   $3x = 21$

Use rectangle RECT and the given information to find each value.



5. m∠RCT = 30°, find m∠ETC. 30°
6. If RC = 5x + 2 and AE = x + 14, find the value of x. $5x + 2 = 2(x + 14)$

   $5x + 2 = 2x + 28$

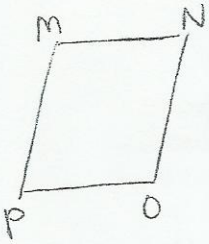
   $3x = 26$

   $x = \frac{26}{3}$
7. If m∠EAC = 40°, find m∠AEC. 180 − 40 = 140

   70°

Determine whether EFGH is a parallelogram, a rectangle, a rhombus, or a square for each set of vertices. State yes or no for each and explain why or why not. Show work to support the explanations.

8. M(1, 5), N(6, 5), O(6, 10), P(1, 10)



slope $\overline{MN} = \frac{5-5}{1-6} = 0$

slope $\overline{PO} = \frac{10-10}{6-1} = 0$ } ||

slope $\overline{MP} = \frac{10-5}{1-1}$ = undef

slope $\overline{NO} = \frac{5-10}{6-6}$ = undef } ||

slope $\overline{MO} = \frac{5-10}{1-6} = \frac{-5}{-5} = 1$

slope $\overline{NP} = \frac{5-10}{6-1} = \frac{-5}{5} = -1$ } ⊥

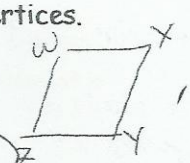
parallelogram: yes - opp sides ||

rectangle: yes - cons. sides ⊥

rhombus: yes - diag. ⊥

square: yes - both rectangle & rhombus

9. W(5, 4), X(3, -6), Y(0, -10), Z(2, 0)



slope $\overline{WX} = \frac{4+6}{5-3} = \frac{10}{2} = 5$

slope $\overline{ZY} = \frac{-10-0}{0-2} = \frac{-10}{-2} = 5$ } ||

slope $\overline{WZ} = \frac{4-0}{5-2} = \frac{4}{3}$

slope $\overline{XY} = \frac{-6+10}{3-0} = \frac{4}{3}$ } ||

slope $\overline{WY} = \frac{4+10}{5-0} = \frac{14}{5}$

slope $\overline{XZ} = \frac{-6-0}{3-2} = -6$ } not ⊥

parallelogram: yes - opp sides ||

rectangle: no - cons. sides not ⊥

rhombus: no - diag. not ⊥

square: no - not rectangle and rhombus

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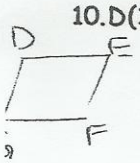
10. D(1, 10), E(-4, 0), F(7, 2), G(12, 12)



slope $\overline{DE} = \frac{10-0}{1+4} = \frac{10}{5} = 2$

slope $\overline{GF} = \frac{2-12}{7-12} = \frac{-10}{-5} = 2$ } ||

slope $\overline{DG} = \frac{10-12}{1-12} = \frac{-2}{-11} = \frac{2}{11}$ } not ⊥

slope $\overline{EF} = \frac{0-2}{-4-7} = \frac{-2}{-11} = \frac{2}{11}$ } ||

slope $\overline{DF} = \frac{10-2}{1-7} = \frac{8}{-6} = -\frac{4}{3}$

slope $\overline{EG} = \frac{0-12}{-4-12} = \frac{-12}{-16} = \frac{3}{4}$ } ⊥

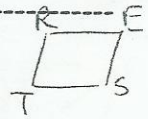
parallelogram: yes opp sides ||

rectangle: no, cons sides not ⊥

rhombus: yes diag. ⊥

square: no, not rectangle

11. R(5, 6), E(7, 5), S(9, 9), T(7, 10)



slope $\overline{RE} = \frac{6-5}{5-7} = \frac{1}{-2}$

slope $\overline{TS} = \frac{10-9}{7-9} = \frac{1}{-2}$ } ||

slope $\overline{RT} = \frac{6-10}{5-7} = \frac{-4}{-2} = 2$ } ⊥

slope $\overline{ES} = \frac{5-9}{7-9} = \frac{-4}{-2} = 2$ } ||

slope $\overline{RS} = \frac{6-9}{5-9} = \frac{-3}{-4} = \frac{3}{4}$

slope $\overline{TE} = \frac{5-10}{7-7}$ = undef. } not ⊥

parallelogram: yes - opp sides ||

rectangle: yes cons. sides ⊥

rhombus: no, diag. not ⊥

square: no - not rhombus

Geometry Worksheet
Rectangles, Squares & Rhombi (6.4)

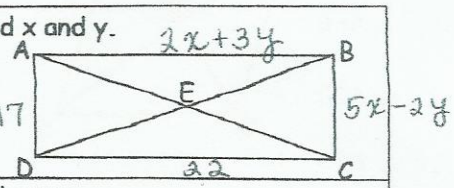
Name_____
Date_____ Period_____

1. In rectangle ABCD, AB = 2x + 3y, BC = 5x - 2y, CD = 22, and AD = 17. Find x and y.

$2(2x+3y=22)$
$3(5x-2y=17)$

$4x+6y=44$
$15x-6y=51$
$19x=95$

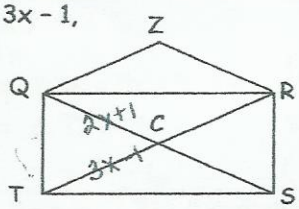
$\boxed{x=5 \quad y=4}$



In the diagram for problems 2-7, QRST is a rectangle and QZRC is a parallelogram.

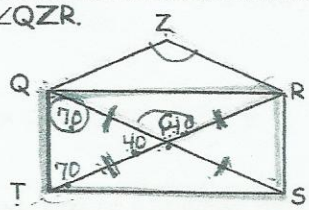
2. If QC = 2x + 1 and TC = 3x - 1, find x.

$2x+1=3x-1$
$\boxed{2=x}$



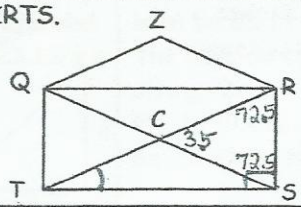
3. If m∠TQC = 70, find m∠QZR.

$m\angle QZR = 140°$



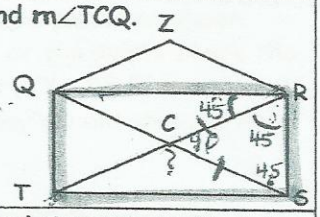
4. If m∠RCS = 35, find m∠RTS.

$m\angle RTS = 17.5°$



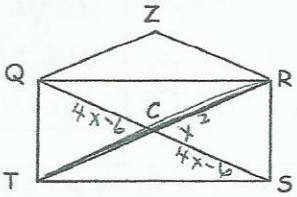
5. If m∠QRT = m∠TRS, find m∠TCQ.

$m\angle TCQ = 90°$



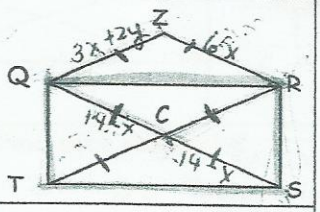
6. If RT = x^2 and QC = 4x - 6, what is the value of x?

$x^2 = 8x - 12$
$x^2 - 8x + 12 = 0$
$(x-2)(x-6) = 0$
$\boxed{x=2 \quad x=6}$



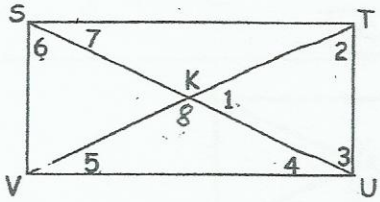
7. RZ = 6x, ZQ = 3x + 2y, and CS = 14 - x. Find the values of x and y. Is QZRC a "special" parallelogram? If so, what kind? $\boxed{\text{it is a rhombus}}$

$6x = 14 - x$
$7x = 14$
$\boxed{x=2} \quad \boxed{y=3}$
$6+2y=12$
$2y=6$



Use rectangle STUV for questions 8-11.

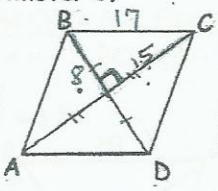
8. If m∠1 = 30, m∠2 = 75°
9. If m∠6 = 57, m∠4 = 33°
*10. If m∠8 = 133, m∠2 = 66.5
11. If m∠5 = 16, m∠3 = 74°



12. ABCD is a rhombus. If the perimeter of ABCD = 68 and BD = 16, find AC.

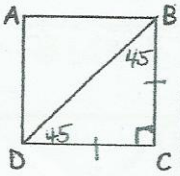
$\frac{68}{4} = 17$ $\boxed{AC = 30}$

$8^2 + x^2 = 17^2$
$x^2 = 225$
$x = 15$

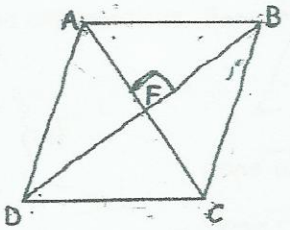


13. ABCD is a square. If m∠DBC = x^2 - 4x, find x.

$x^2 - 4x = 45$
$x^2 - 4x - 45 = 0$
$(x-9)(x+5) = 0$
$\boxed{x=9 \quad x=-5}$



Use rhombus ABCD for problems 14-19



14. If m∠BAF = 28, m∠ACD = 28.

15. If m∠AFB = 16x + 6, x = 5.25. $16x+6=90$ $16x=84$

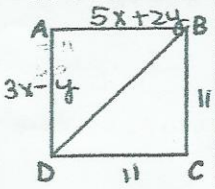
16. If m∠ACD = 34, m∠ABC = 112.

17. If m∠BFC = 120 − 4x, x = 7.5. $120-4x=90$ $30=4x$

18. If m∠BAC = 4x + 6 and m∠ACD = 12x − 18, x = 3.
$4x+6=12x-18$
$24=8x$

19. If m∠DCB = x^2 − 6 and m∠DAC = 5x + 9, x = 12
$2(\angle DAC)=\angle DCB$ $x^2-10x-24=0$ $x=12$ or $x=-2$
$10x+18=x^2-6$ $(x-12)(x+2)=0$

20. ABCD is a square. AB = 5x + 2y, AD = 3x − y, and BC = 11. Find x and y. $y=3x-11$

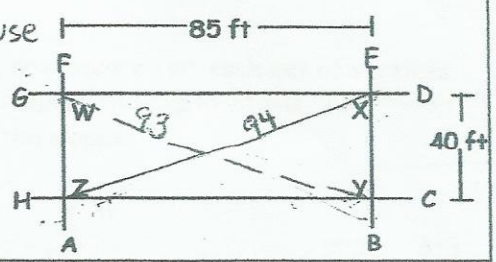


$5x+2y=11$
$2(3x-y=11)$
$6x-2y=22$
$11x=33$
$\boxed{x=3}$
$15+2y=11$
$4=-2y$
$\boxed{-2=y}$

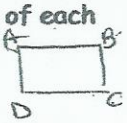
21. A contractor is measuring for the foundation of a building that is to be 85 ft by 40 ft. Stakes and string are placed as shown. The outside corners of the building will be at the points where the strings cross. He then measures and finds WY = 93 ft and XZ = 94 ft. Is WXYZ a rectangle? If not, which way should stakes E and F be moved to made WXYZ a rectangle?

not rectangle because diagonals are not ≅

move left



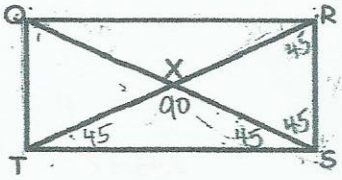
22. ABCD is a rectangle. Find the length of each diagonal if AC = 2(x − 3) and BD = x + 5.



$2x-6=x+5$
$\boxed{x=11}$ $\boxed{AC=BD=16}$

23. ABCD is a rectangle. Find each diagonal if $AC=\frac{3c}{9}$ and BD = 4 − c.
$\frac{3c}{9}=4-c$ $3c=36-9c$
$12c=36$
$\boxed{c=3}$ $\boxed{diag=1}$

Given rectangle QRST



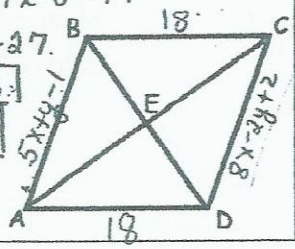
120° 24. If $\overline{RX}\cong\overline{QT}$, find m∠TXS.

6.5 25. If m∠RQS = 30° and QS = 13, find SR.

6.2 26. If m∠QST = 45° and QT = 6.2, find QR.

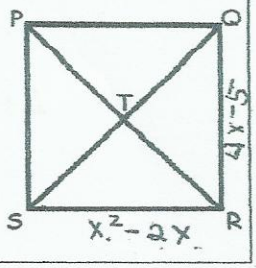
27. Given rhombus ABCD, AB = 5x + y − 1, BC = 18, CD = 8x − 2y + 2. Find x and y.

$5x+y-1=18$
$*5x+y=19$
$8x-2y+2=18$
$8x-2y=16$
$*4x-y=8$
$5x+4x-8=19$
$9x=27$
$\boxed{x=3}$
$\boxed{y=4}$

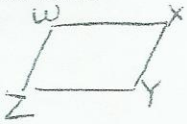


28. Given square PQRS, SR = x^2 − 2x, QR = 4x − 5. Find x, SR, and QR.

$x^2-2x=4x-5$
$x^2-6x+5=0$
$(x-5)(x-1)=0$
$\boxed{x=5}$, $\cancel{x=1}$
$\boxed{SR=QR=15}$



Determine whether WXYZ is a parallelogram, a rectangle, a rhombus, or a square for each set of vertices. State yes or no for each and explain why or why not. Show work to support the explanations. For example, if you say the sides are parallel then you need to calculate the slopes.



29. W(5, 6), X(7, 5), Y(9, 9), Z(7, 10)

Parallelogram: opp sides ||

Rectangle: cons. sides ⊥

Rhombus: no- diag not ⊥

Square: no - not rhombus

slope $\overline{WX} = \frac{6-5}{5-7} = \frac{1}{-2}$ } ||

slope $\overline{ZY} = \frac{9-10}{9-7} = -\frac{1}{2}$

slope $\overline{WZ} = \frac{6-10}{5-7} = \frac{-4}{-2} = 2$ } ||

slope $\overline{XY} = \frac{5-9}{7-9} = \frac{-4}{2} = 2$

⊥

slope $\overline{WY} = \frac{6-9}{5-9} = \frac{3}{4}$

slope $\overline{XZ} = \frac{5-10}{7-7} =$ undef

30. W(-3, -3), X(1, -6), Y(5, -3), Z(1, 0)

Parallelogram: yes - opp sides ||

Rectangle: no - cons sides not ⊥

Rhombus: yes - diag. ⊥

Square: no – not rectangle

slope $\overline{WX} = \frac{-3+6}{-3-1} = -\frac{3}{4}$ } ||

slope $\overline{ZY} = \frac{-3-0}{5-1} = -\frac{3}{4}$

slope $\overline{WZ} = \frac{-3-0}{-3-1} = \frac{3}{4}$

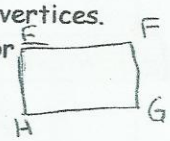
slope $\overline{XY} = \frac{-6+3}{1-5} = \frac{-3}{-4} = \frac{3}{4}$ } ||

not ⊥

slope $\overline{WY} = \frac{-3+3}{-3-5} = 0$

slope $\overline{XZ} = \frac{-6-0}{1-1} =$ undef

Determine whether EFGH is a parallelogram, a rectangle, a rhombus, or a square for each set of vertices. State yes or no for each and explain why or why not. Show work to support the explanations. For example, if you say the sides are parallel then you need to calculate the slopes.



31. E(0, -3), F(-3, 0), G(0, 3), H(3, 0)

Parallelogram: yes - opp sides ||

Rectangle: yes - cons. sides ⊥

Rhombus: yes - diag. ⊥

Square: yes - both rect. & rhombus

slope $\overline{EF} = \frac{-3-0}{0+3} = -1$ } ||

slope $\overline{HG} = \frac{3-0}{0-3} = \frac{3}{-3} = -1$

slope $\overline{EH} = \frac{-3-0}{0-3} = \frac{-3}{-3} = 1$ } ||

slope $\overline{FG} = \frac{0-3}{-3-0} = \frac{-3}{-3} = 1$

⊥

slope $\overline{EG} = \frac{-3-3}{0-0} =$ undef

slope $\overline{FH} = \frac{0-0}{-3-3} = 0$

32. E(2, 1), F(3, 4), G(7, 2), H(6, -1)

Parallelogram: yes-opp sides ||

Rectangle: no - cons. sides not ⊥

Rhombus: no - diag not ⊥

Square: no - not rectangle or rhombus

slope $\overline{EF} = \frac{1-4}{2-3} = \frac{-3}{-1} = 3$ } ||

slope $\overline{HG} = \frac{2+1}{7-6} = \frac{3}{1} = 3$

slope $\overline{EH} = \frac{1+1}{2-6} = \frac{2}{-4} = -\frac{1}{2}$ } ||

slope $\overline{FG} = \frac{4-2}{3-7} = \frac{2}{-4} = -\frac{1}{2}$

not ⊥

slope $\overline{EG} = \frac{1-2}{2-7} = \frac{-1}{-5} = \frac{1}{5}$ } not ⊥

slope $\overline{FH} = \frac{4+1}{3-6} = \frac{5}{-3}$