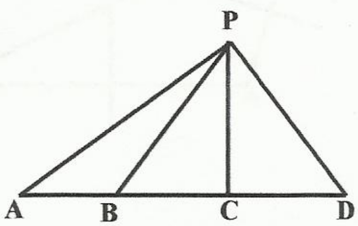
Geometry
Section 5.3

Name KEY
Date ____________ Period ________

Medians, Altitudes, Perpendicular Bisectors

Four special types of segments are associated with triangles.

- A **median** is a segment that connects a vertex of a triangle to the midpoint of the opposite side.
- An **altitude** is a segment that has one endpoint at a vertex of a triangle and the other endpoint on the line containing the opposite side so that the altitude is perpendicular to that line.
- An **angle bisector** of a triangle is a segment that bisects an angle of the triangle and has one endpoint at the vertex of that angle and the other endpoint on the side opposite that vertex.
- A **perpendicular bisector** is a segment or line that passes through the midpoint of a side and is perpendicular to that side.



Complete using the figure at the right.

$\overline{BP}$ 1. If AB = BC, then $\overline{BP}$ is a median of ΔAPC.

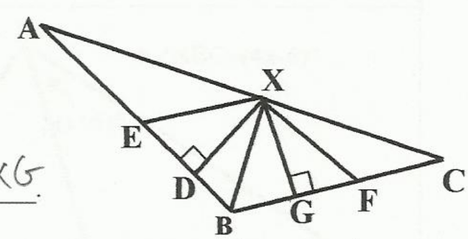
$\overline{BD}$ 2. If $\overline{PC}$ is a perpendicular bisector of $\overline{BD}$ then BC = DC.

$\overline{PD}$, $\overline{PA}$ 3. If ∠APD is a right angle, then $\overline{PD}$ and $\overline{PA}$ are altitudes of ΔAPD.

AC = CD 4. If $\overline{PC}$ is a median of ΔPBD, then AC = CD.

$\overline{PC}$, $\overline{BD}$ 5. If BC = CD and $\overline{PC} \perp \overline{BD}$, then $\overline{PC}$ is a perpendicular bisector of $\overline{BD}$

∠PCA 6. If $\overline{PC}$ and $\overline{AC}$ are both altitudes of ΔPCA, then ∠PCA is a right angle.



Complete.

∠ABX ≅ ∠CBX, XG 7. If $\overline{BX}$ bisects ∠ABC, then ∠ABX ≅ ∠CBX and DX = XG.

DB, XB 8. If $\overline{DX}$ is the perpendicular bisector of $\overline{EB}$, then ED = DB and XE = XB.

$\overline{XG}$, ∠XFB 9. If XB = XF, then XG is the perpendicular bisector of $\overline{BF}$, and ∠XBF ≅ ∠XFB

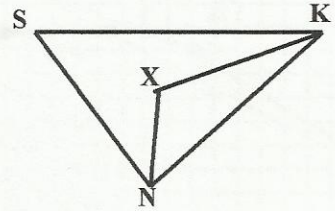
∠ABC 10. If XD = XG, then XB is the bisector of ∠ABC.

Complete the statement.

$\overline{SK}$, $\overline{NK}$ 11. If X is on the bisector of $\angle SKN$, then X is equidistant from $\underline{SK}$ and $\underline{NK}$

$\overline{NK}$, $\overline{NS}$ 12. If X is on the bisector of $\angle SNK$, then X is equidistant from $\underline{NK}$ and $\underline{NS}$.

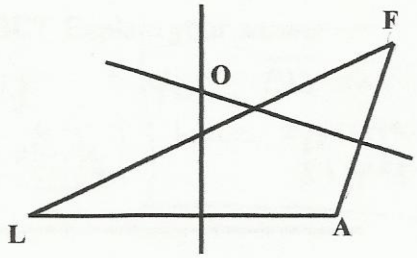
Angle bisector of $\angle KSN$ 13. If X is equidistant from $\overline{SK}$ and $\overline{SN}$, then X lies on the $\underline{\text{angle bisector of } \angle KSN}$



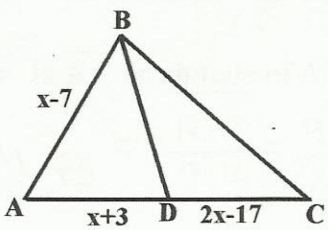
L, A 14. If O is on the perpendicular bisector of $\overline{LA}$, then O is equidistant from $\underline{L}$ and $\underline{A}$.

F, A 15. If O is on the perpendicular bisector of $\overline{AF}$, then O is equidistant from $\underline{F}$ and $\underline{A}$.

$\perp$ bisector of $\overline{LF}$ 16. If O is equidistant from L and F, then O lies on the ____.



17. Find AB if $\overline{BD}$ is a median of ΔABC.



$x+3=2x-17$

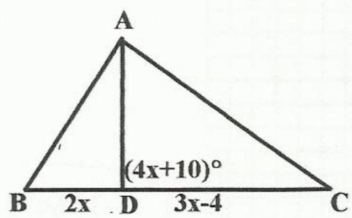
$-x=-20$

$x=20$

$x-7$

$(20)-7=\boxed{13}$

18. Find BC if $\overline{AD}$ is an altitude of ΔABC.



$4x+10=90$

$4x=80$

$x=20$

$2(20)+3(20)-4$

$40+60-4$

$\boxed{96}$

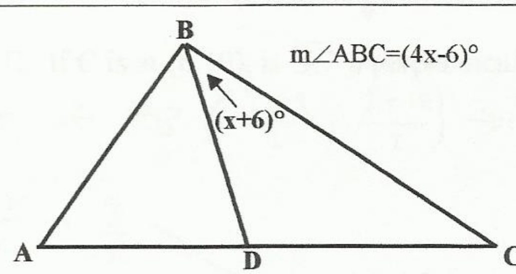
19. Find $m\angle ABC$ if $\overline{BD}$ is an angle bisector of ΔABC.

$4x-6=2(x+6)$

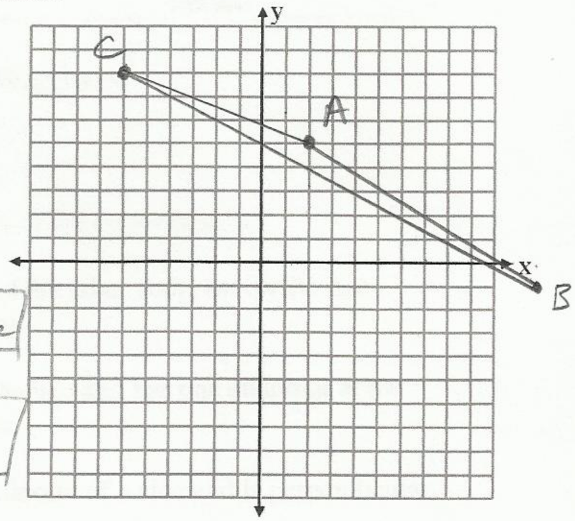
$4x-6=2x+12$

$2x=18$

$\boxed{x=9}$



In problems 20-23, A(2,5), B(12,-1), and C(-6,8) are the vertices of ΔABC.



20. What are the coordinates of K if $\overline{CK}$ is a median of ΔABC?

midpoint of $\overline{AB} \rightarrow \left(\frac{2+12}{2}, \frac{5-1}{2}\right) \rightarrow \boxed{(7,2)}$

21. What is the slope of the perpendicular bisector of $\overline{AB}$?

Slope of $AB = \frac{-1-5}{12-2} = \frac{-6}{10} = -\frac{3}{5}$

$\boxed{\perp \text{ bisector} \rightarrow \frac{5}{3} \text{ slope}}$

22. What is the slope of $\overline{CL}$ if $\overline{CL}$ is the altitude from point C?

Slope of $AB = -\frac{3}{5}$ $\boxed{\text{Altitude from } C = \frac{5}{3} \text{ slope}}$

23. Point N on $\overline{BC}$ has coordinates $\left(\frac{8}{5}, \frac{21}{5}\right)$. Is $\overline{NA}$ an altitude of ΔABC? Explain your answer.

$A(2,5)$ $N(1.6, 4.2)$ $C(-6,8)$ $B(12,-1)$

$m_{\overline{AB}} = \frac{-0.8}{-0.4} = \boxed{2}$ $m_{\overline{CB}} = \frac{-1-8}{12+6} = \frac{-9}{18} = \boxed{-\frac{1}{2}}$

Yes, $\overline{AN}$ and $\overline{CB}$ have opposite reciprocal slopes

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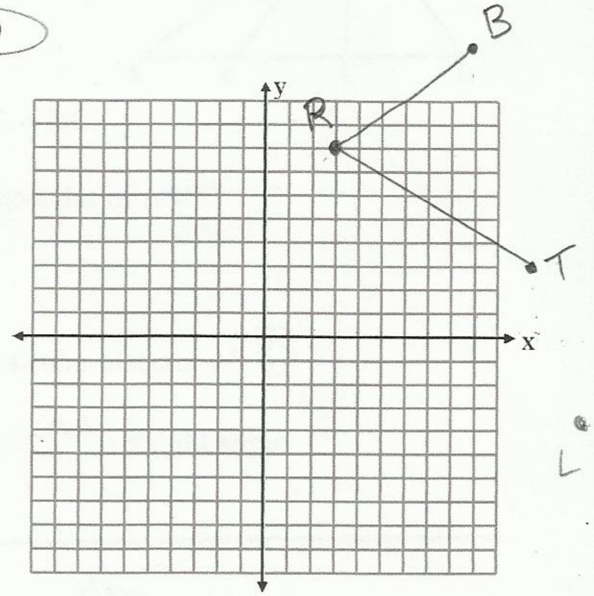
24. $\overline{RT}$ is a median in ΔRLB with points R(3,8), T(12,3), and B(9,12)

midpoint

a. What are the coordinates of L?



$\left(\frac{x+9}{2} = 12, \frac{y+12}{2} = 3\right)$ $\boxed{L(15,-6)}$

$(x+9=24, y+12=6)$

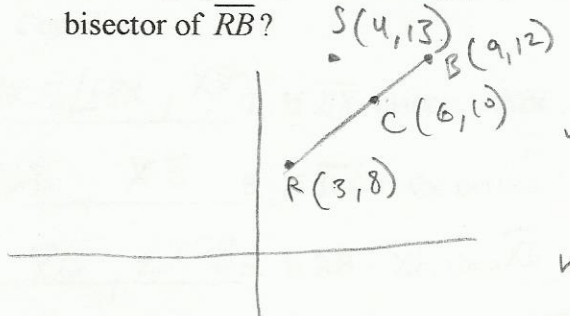
b. Is $\overline{RT}$ an altitude of ΔRLB?

$m_{\overline{TB}} = \frac{12-3}{9-12} = \frac{9}{-3} = \boxed{-3}$

$m_{\overline{RT}} = \frac{3-8}{12-3} = \boxed{\frac{-5}{9}}$

NO, slopes are not $\perp$

c. The graph of point S is at (4,13). $\overline{SC}$ intersects $\overline{RB}$ at C. If C is at (6,10), is $\overline{SC}$ a perpendicular bisector of $\overline{RB}$?



✓ midpoint of RB $\left(\frac{3+9}{2}, \frac{8+12}{2}\right) \rightarrow (6,10)$

✓ $m_{SC} = \frac{13-10}{4-6} = \frac{3}{-2}$

✓ $m_{RB} = \frac{12-8}{9-3} = \frac{4}{6} = \frac{2}{3}$

$\perp$ slopes

$\boxed{\text{YES}}$