

Valid/Invalid Reasoning & Error Analysis

You already know that the original conditional and its contrapositive are logically equivalent statements. We will use this to rewrite sentences so that we can apply the law of syllogism.

Example:

If you can play the banjo, then you have practiced very hard.

If you can't play "Oh Susannah," then you haven't practiced very hard.

In symbols these two statements are:

$p \rightarrow q$ If you can play the banjo, then you have practiced very hard.
 $\sim r \rightarrow \sim q$ If you can't play "Oh Susannah," then you haven't practiced very hard.

Replace the second statement with its contrapositive (because it is logically equivalent to it).

$p \rightarrow q$ If you can play the banjo, then you have practiced very hard.
 $q \rightarrow r$ If you have practiced very hard, then you can play "Oh Susannah."

Apply the law of syllogism to conclude:

$p \rightarrow r$ If you can play the banjo, then you can play "Oh Susannah."

NOTE: You can only replace a sentence with its contrapositive!

Write each sentence in "if-then" form (or using symbols or key words). Replace a sentence with its contrapositive if needed. Also, switch the two sentences if needed. Then write either the conclusion statement or "no conclusion possible."

1. If your children can copy their term papers, they will have more time to watch television. *copy* → TV
 If you buy the Encyclopedia Cribanana, your children can copy their term papers from it. *encyclopedia* → *copy*
encyclopedia → TV : If you buy the Encyclopedia, then your children will have more time to watch TV.
conclusion →
2. If a lawn is made of AstroTurf, then it is always green. *Astro* → green
 A lawn is always green if it gets plenty of water. *water* → green
no conclusion
3. If your name is in Who's Who, then you know what's what. *who* → *what*
 If you're not sure of where's where, then you don't know what's what. *~ where* → *~ what*
Conclusion: If your name's in who's who, then you're sure of where's where. *what's where* (contrapositive)
4. If you are afraid of earthquakes, then you shouldn't live in California. *afraid* → *~ CA*
 If you don't mind getting shaken up, then you are not afraid of earthquakes. *shaken up* → *~ afraid*
no conclusion
5. If you can leap tall buildings in a single bound, then you are a powerful jumper. *buildings* → *jumper*
 If you are not a powerful jumper, then you're not from Calaveras County. *~ jumper* → *~ calaveras*
no conclusion
6. If you read Mewsweek magazine, then you like cats. *mewsweek* → *cats*
 If you are fond of mice, then you do not like cats. *mice* → *~ cats*
If you read mewsweek, then you aren't fond of mice. *cats* → *~ mice* (contrapositive)
7. All white flowers are fragrant. *white* → *fragrant*
 All carnations are white flowers. *carnations* → *white*
Conclusion: If a flower is a carnation, then it is fragrant.
 OR All carnations are fragrant.

8. All shoplifters are dishonest. $\text{shoplifter} \rightarrow \text{dishonest}$
No dishonest person is trustworthy. $\text{dishonest} \rightarrow \sim \text{trustworthy}$

Conclusion: If you're a shoplifter, then you're not trustworthy.

9. All animals get hungry. $\text{animal} \rightarrow \text{hungry}$
No hard workers are hungry. $\text{hard worker} \rightarrow \sim \text{hungry}$ $\text{hungry} \rightarrow \sim \text{hard worker}$

Conclusion: Animals are not hard workers.

10. Every scientist is boring. $\text{scientist} \rightarrow \text{boring}$ Conclusion: Scientists are not mathematicians.
No mathematician is boring. $\text{mathematician} \rightarrow \sim \text{boring}$
 $\text{boring} \rightarrow \sim \text{mathematician}$

Determine if the third statement follows from the first two. If it does, write valid and give the pattern of reasoning (original or contrapositive). If it does not, write invalid and name the error (converse or inverse error). \star Converse and inverse do not have same truth value as original.

1. If the stones are rolling, they are not gathering moss. $\text{rolling} \rightarrow \sim \text{moss}$
If the stones are not gathering moss, they are a smooth group of rocks. $\sim \text{moss} \rightarrow \text{smooth}$
Therefore, if the stones are rolling, they are a smooth rock group.

Valid; original

2. If a penny has an Indian head on it, it is very old. $\text{Indian} \rightarrow \text{old}$
If a penny has an Indian head on it, it is worth more than one cent. $\text{Indian} \rightarrow \text{more than 1 cent}$
Therefore, if a penny is very old, it is worth more than one cent.

not valid; converse error

3. If you walk under a coconut tree, you will probably be hit on the head. $\text{coconut tree} \rightarrow \text{hit}$
If you visit Hawaii, then you will walk under coconut trees. $\text{Hawaii} \rightarrow \text{coconut tree}$
Therefore, if you visit Hawaii, you will probably be hit on the head.

valid; original

4. Blondes have more fun. $\text{blond} \rightarrow \text{more fun}$
A girl has more fun if she is saucy. $\text{saucy} \rightarrow \text{more fun}$
Therefore, if a girl is a blonde, she is saucy.

invalid; converse error

5. Tom would be a gardener if he had a green thumb. ~~green thumb~~ $\text{green thumb} \rightarrow \text{gardener}$
If Tom Thumb were a gardener, he would raise bonsai trees. $\text{gardener} \rightarrow \text{bonsai}$
Therefore, if Tom had a green thumb, he would raise bonsai trees.

Valid; original

6. All donkeys have long ears. $\text{donkeys} \rightarrow \text{long ears}$
All long-eared creatures are habitual eavesdroppers. $\text{long ears} \rightarrow \text{eavesdroppers}$
Therefore, all donkeys are habitual eavesdroppers.

Valid; original

7. All movies directed by Alfred Hitchcock have suspenseful plots. $\text{Hitchcock} \rightarrow \text{suspense}$
The movie North by Northwest has a very suspenseful plot. $\text{N by NW} \rightarrow \text{suspense}$
Therefore, North by Northwest is a Hitchcock movie.

invalid; converse error