1st

KINDS OF REASONING

Some are Better than Others!

Reasoning patterns are part of everyday life. Sometimes you just "know" something. At other times, you may observe patterns and base a conclusion on that. Then, there are times when you are absolutely certain that an event will occur.

Definitions:

1. **Intuitive Reasoning** – Reasoning by using beliefs / hunches. Intuitive thinking involves "sensing that something is true and "just feeling sure" that you are correct. It is "jumping to a conclusion" without any real evidence.

Example: Mrs. Jones doesn't want her son to go to the movies because she just "has a feeling" he will get himself into trouble.

2. **Inductive Reasoning** – Reasoning by finding a general principle based upon the evidence. It is an "educated guess" based on data or observations. Cases include making decisions on the basis of polls, drawing a conclusion from a computer lab investigation, and making decisions based on observations in science labs. Since it is not possible to examine every situation, there is always the possibility that a contradiction will be found.

Example: After asking the ages of 25 freshmen, Judy reasoned that all freshmen are at least 13 years old.

3. **Deductive Reasoning:** Building a logical argument / explanation by making conjectures based on established facts– reasoning without any guessing. The conclusion is absolutely certain – there is no room for doubt. New facts are deduced from accepted facts.

Using 'If – Then' statements to make conjectures

Example: Today is Tuesday so tomorrow must be Wednesday.

Tell whether the process used is <u>intuitive</u>, <u>inductive</u>, <u>deductive</u>, or <u>none</u> of these.

1. The first four times Susan ate peanuts, she became ill. Her mother decided: "Peanuts are bad for Susan."

   inductive

2. A girl is fifteen years old. She points out: "Three years from now, I'll be eighteen.

   deductive

3. A boy remarks: If a number is greater than ten, then twice that number is greater than twenty.

   deductive

4. A courtroom spectator merely looks into a defendant's eyes before saying, "He's guilty, I tell you.

   intuitive

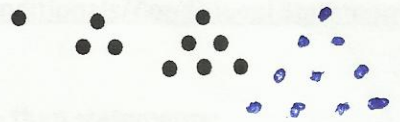
5. A juror serves a case in which the charge is speeding. Upon finding out that the defendant has already been convicted several times for speeding, a juror mutters to himself: "That speeder is guilty."

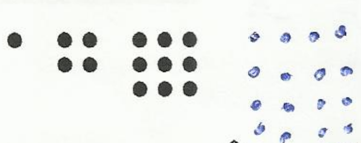
   inductive

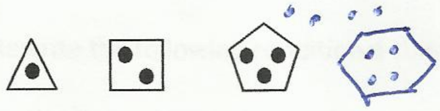
Inductive Reasoning—Finding Patterns

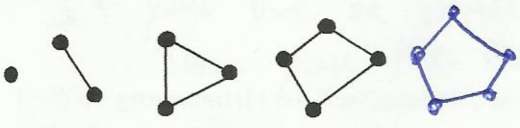
I. Picture Patterns

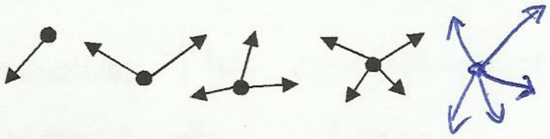
Draw the next shape in each picture pattern.

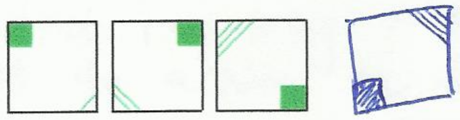
1. 

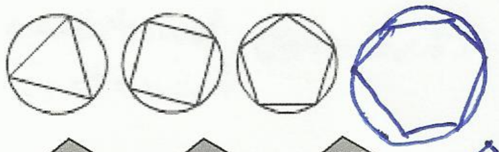
2. 

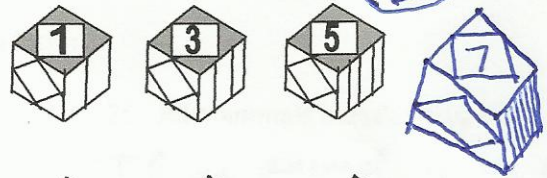
3. 

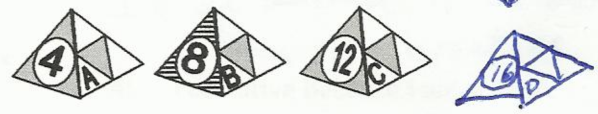
4. 

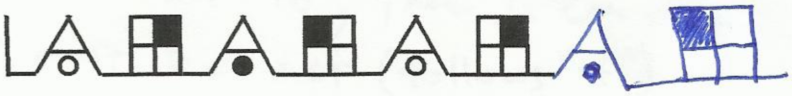
5. 

6. 

7. 

8. 

9. 

10. 

II. Number Patterns

Use inductive reasoning to find the next term of each sequence.

11. 1, 2, 4, 8, 16, 32, 64

2 3 4 5 6 +7

12. 1, 3, 6, 10, 15, 21, 28

4 9 16 25 36 49

13. 2, 6, 15, 31, 56, 92, 141

92
49
141

14. T, Q, N, K, H, E, B

15. $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$, 5/6

III. Sequence Patterns

Which rule can be used to find the next number, or any number, in the sequence?

n: 1 2 3 4 5

16. 0, 1, 3, 6, 10...

A) $\frac{n(2n-2)}{2}$ B) $\frac{n(n-1)}{2}$

C) $\frac{(n-1)}{2}$ D) $2(n+1)-4$

17. 4, 7, 10, 13...

A) $4n$ B) $n+3$

C) $2n-1$ D) $3n+1$

n: 1 2 3 4

18. y: 1, 3, 9, 27...

A) $(n-1)^{n-1}$ B) $(2n-1)^2$

C) $(3)^{n-1}$ D) n^2

19. 8, 12, 16, 20, 24...

A) n^2+7 B) $2(n^2+3)$

C) $4(n+1)$ D) $-2(n-1)+8n$

20. 12, 14, 16, 18...

A) $12n$ B) $2n+10$

C) $5n+2$ D) $12n-5$

Conditional Statements

Conditionals/Conditional Statements: a statement requiring a condition to be met to achieve the desired result.

If – then statements: a form of conditional statements used to clarify statement.

Rewrite the following conditional statements in If-Then form:

1: You live in Houston and, thus, you live in Texas

If you live in Houston, then you live in TX.

2: All mammals breathe oxygen.

If an animal is a mammal, then it breathes oxygen.

3: I will be grounded if I fail the Geometry test.

If I fail ... then ... grounded.

4: I can drive because I am sixteen.

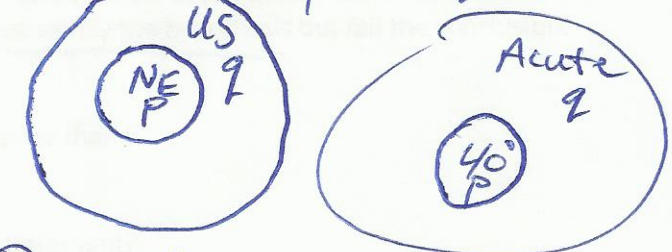
If 16, then I can drive

Hypothesis (p): the condition that must be met (follows "if")

Conclusion (q): the desired result.

(19) New England → United States

p → q



(20) 40° → Acute

p → q

Example: Determine the hypothesis and conclusion.
"If it is Tuesday, then Phil plays tennis."

p Hypothesis: It is Tuesday

q Conclusion: Phil plays tennis

Symbols: $p \rightarrow q$

"if p" "then q"

***Other Words** that help identify the Hypothesis and Conclusion*

Some synonyms for "if"	Some synonyms for "then"
when	only if
because	so
provided that	therefore
since	thus
after	implies
due to the fact	in order to
in the event	to have been
as a result	caused
on account of	before

Rewrite each of the following conditionals in if-then form.
Then underline the hypothesis and circle the conclusion.

1. All right angles are congruent. If angles are right, then they are congruent
2. You can play when you finish your homework.
   If ... homework, then ... play.
3. Since you did your chores you will get your allowance.
   If you did chores ..., then ... allowance
4. Sally is grounded because she broke curfew.
   If Sally broke curfew, then she will be grounded.
5. You can legally drive provided you have a driver's license.
   If you have driver's license, then ... legally drive.
6. Mom saves money if she clips coupons.
   If mom clips ... then she saves $
7. Getting a good grade makes Jimmy happy.
   If Jimmy gets a good grade, then he will be happy.
8. Because you didn't turn in your homework you will get a zero.
   If you don't turn in HW, then you will get a zero.
9. When you are happy you will do well.
   If you are happy, then you will do well.
10. Jill will use Cliff notes because she didn't read the book.
    If Jill doesn't read ... then ... Cliff notes

Not all conditional statements are true. It takes only one false example to show that a conditional is false. The false example is called a ***counterexample***. A counterexample must satisfy the hypothesis but fail the conclusion.

Example 1: If a number is greater than 3, then the number is greater than 5.
Which, if any, of these numbers is an example?
2, 4, 5, or 6
Which, if any, of these numbers is a counterexample to that statement?
4, 5

Example 2: If a number is greater than 5, then the number is greater than 3.
Which, if any, of these numbers is an example?
2, 4, 5, or 6
Which, if any, of these numbers is a counterexample to the statement?
none

Are these conditionals true or false? If false, give a counterexample.

1. If Jane is your sister, then she is older than you. F → Jane could be younger
2. If you are in geometry, then you are a sophomore. F → Freshmen in Geometry
3. If you are a sophomore, then you are in geometry. F → Sophomore in Algebra 2.
4. If your clothes are wet, then it is raining. F → sprinkler
5. If $m\angle E = 36$, then $\angle E$ is an acute angle. T
6. If $\angle E$ is an acute angle, then $m\angle E = 36$. F → 40° angle
7. If two angles are right angles, then they have the same measure. T
8. If two angles have the same measure, then they are right angles. F → two 80° angles.

Converse: Created by switching hypothesis + conclusion

Example: Write the converse of "If you live in Houston, then you live in Texas."

hyp. (p) conclusion (q)

Converse: If ... TX → Houston
(q) (p)

Symbols:

$q \to p$ TX → Houston

Example: Write the converse of the true conditional: "Acute angles have measures less than 90 degrees."

→ If $< 90°$, then ... acute.

Biconditional Statements

When a conditional and its converse are true, you can combine them as a true biconditional. This is the statement you get by connecting the conditional and its converse with the phrase "if and only if".

A biconditional combines $p \to q$ and $q \to p$ as $p \leftrightarrow q$.

Example:

Consider the true conditional statement. Write its converse.

Conditional: If an angle is right, then its measure is 90 degrees.

Converse: If an angle measures 90°, then it is right

Is the converse true? If it is, combine the statements as a biconditional.

Biconditional: An angle is a right angle, if and only if its measure is 90°

Example:

Consider the true conditional statement. Write its converse.

Conditional: If $x = 5$, then $x + 15 = 20$.

Converse: If $X + 15 = 20$, then $X = 5$

Is the converse true? If it is, combine the statements as a biconditional.

Biconditional: $X + 15 = 20$ if and only if $X = 5$

A good definition is one that is reversable That means you can write a good definition as a biconditional.

$m\angle A = 90°$

Example: Lines are perpendicular if and only if they intersect to form right angles.

Conditional: If lines are $\perp$, then they intersect to form rt. $\angle$s.

Converse: If lines intersect to form rt. $\angle$s, then they are $\perp$.

Section 2.3 Deductive Reasoning

1. ***Deductive Reasoning:*** Building a LOGICAL argument / explanation by making conjectures based on established facts

Using 'If – Then' statements to make conjectures

2. ***Law of Detachment:*** If p → q is a true conditional and p is true, then q is true.

Example:

hyp- conclusion

Conditional: If a vehicle is a car, then it has four wheels

p q

Given: A sedan is a car

Conclusion: A Sedan has four wheels

Is the CONVERSE of this law true? NO counter examples: wagon, four wheeler.

Example:

Conditional: If it rains, John will get wet.

Given: John is wet

Conclusion: Can we conclude it rained? NO

Counter examples: swimming, sprinklers, etc.

*** ***Knowing the "result" did occur does NOT guarantee the condition was met***

conclusion hypothesis

Example: If two numbers are odd, then their sum is even is a true conditional. 3 and 5 are odd numbers. Use the Law of Detachment to reach a logical conclusion.

satisfied

Conditional: If two numbers are odd, then their sum is even

p

Given: 3 and 5 are odd numbers.

Conclusion: The sum of 3+5 is even.

3. *Law of Syllogism:* If $p \rightarrow q$ and $q \rightarrow r$ are true conditionals, then $p \rightarrow r$ is also true.

IF $p \rightarrow q$
AND $q \rightarrow r$
THEN $p \rightarrow r$

$p \rightarrow q \rightarrow r$

$p \rightarrow r$

Example:

Conditionals: If you live in Houston, then you live in Texas.
If you live in Texas, then you live in the United States.

Conditional: If you live in Houston, then ... US.

Example: Can any two statements be "Linked" together?

Conditionals: If you watch TV, then you have less time to read.
If you watch TV, then you are lazy.

Can you make a conclusion? no conclusion

Determine if a valid conclusion can be reached from the two true statements using the Law of Syllogism or the Law of Detachment. If a valid conclusion is possible, state it and the law that is used. If a valid conclusion does not follow, write no conclusion.

1.) Angles that are complementary have measures with a sum of 90 degrees.
Angle A and angle B are complementary.

Angles A and B have measures with a sum of 90°. (Detachment)

2.) If a student gets an A on a final exam, then the student will pass the course.
Alex scored a 93 on the final exam.

Alex will pass the course. (Detachment)

3.) (A) If you have a dog, then you are a pet owner.
(B) If you are a pet owner, then you have to take care of your pet.

If you have a dog, you have to take care ...

4.) A student who wants to go to college must study hard.
Melissa studies hard.

No conclusion

5.) (A) If I go to the mall, I will spend too much money.
(B) If I go on vacation, I will spend too much money.

no conclusion

Section 5.4: Inverses, Contrapositives/ Indirect Reasoning

Negation: gives a statement the opposite meaning.

Statement (p) — Negation ($\sim p$)

It is raining — It is not raining.

Inverse: negation of hypothesis and conclusion.

Example: Write the inverse of "If you live in Houston, then you live in Texas."

p → q

Inverse: If you don't live in Houston, then you don't live in TX

Symbols: $\sim p \rightarrow \sim q$

$\sim$ Houston $\rightarrow \sim$ TX

Example: Write the inverse of the true conditional: "Right angles are congruent." Determine if the inverse is true or false.

Steps:

1. If-Then statement: If angles are right angles, then they are $\cong$.
2. Inverse: If angles aren't right angles, then they aren't $\cong$.
3. True/False, example: False; two 30° angles aren't right angles, but they are congruent.

Contrapositive: created by switching and negating hypothesis + conclusion.

Example: Write the contrapositive of "If you live in Houston, then you live in Texas.".

p → q

Contrapositive: If you do not live in TX, then you do not live in Houston.

Symbols: $\sim q \rightarrow \sim p$ or $\sim$ TX $\rightarrow \sim$ Houston

Example: Write the contrapositive of the true conditional: "Parallel lines never intersect." Determine if the contrapositive is true or false.

If lines are $\parallel$, then they never intersect.

Steps:

1. Original statement true/false: true
2. Contrapositive: If lines intersect, then they are not $\parallel$.
3. True/False:

★ The contrapositive has the same truth value as the original statement, thus it can be used in place of the original statement in forming a logical argument.

Write the original statement in "if-then" form. Then write its converse, inverse, and contrapositive.

Example:

odd → ~ even

Original: An odd number cannot be an even number.

p → q

If-then form of the original: If a number is an odd number, then it cannot be an even number.

Converse of the original: If a number is not an even number, then it is an odd number. ~even → odd

Inverse of the original: If a number is not an odd number, then it can be an even number. ~odd → even

Contrapositive of the original: If a number is an even number, then it is not an odd number. even → ~odd

1. Original: All right angles are congruent.

T If-then form of the original: If angles are right angles, then they are congruent.

F Converse of the original: If angles are congruent, then they are right angles.

F Inverse of the original: If angles are not right angles, then they are not ≅

T Contrapositive of the original: If angles are not congruent, then they are not right angles.

2. Original: Every equilateral triangle is also an isosceles triangle.

If-then form of the original: If a triangle is equilateral, then it is also isosceles.

Converse of the original: If a triangle is isosceles, then it is also equilateral.

Inverse of the original: If a triangle is not equilateral, then it is not isosceles.

Contrapositive of the original: If a triangle is not isosceles, then it is not equilateral.

3. Original: Upon earning an 86%, I will give you a B.

If-then form of the original: If you earn an 86, then I will give you a B.

Converse of the original: If I give you a B, then you have earned an 86.

Inverse of the original: If you don't earn an 86, then I will not give you a B.

Contrapositive of the original: If I don't give you a B, then you have not earned a B.

4. Original: Since you did your chores you will get your allowance.

If-then form of the original: If you did your chores, then you will get your allowance

Converse of the original: If you get your allowance then you did your chores.

Inverse of the original: If you didn't do your chores, you will not get allowance.

Contrapositive of the original: I you don't get your allowance, then you didn't do your chores.

Following each of the numbered statements below are three lettered statements. Identify the relationship of each of the lettered statements to the numbered statement. Write "original", "converse", "inverse", "contrapositive", or "none" as appropriate. Hint: Write each statement in "if-then" form.

1. If you live in Atlantis, then you need a snorkel.

Atlantis → snorkel

inverse a) If you do not live in Atlantis, then you do not need a snorkel. ~Atlantis → ~snorkel

converse b) If you need a snorkel, then you live in Atlantis. snorkel → Atlantis

contrapositive c) If you do not need a snorkel, then you do not live in Atlantis. ~snorkel → ~Atlantis

2. All children like pie. child → pie

converse a) If someone likes pie, then he is a child. pie → child

none b) If someone is not a child, he likes pie. ~child → pie

contrapositive c) A person who does not like pie is not a child. ~pie → ~child

3. Lady kangaroos do not need handbags. lady kangaroo → ~handbag

inverse a) If a kangaroo is not a lady, it needs a handbag. ~lady → handbag

contrapositive b) If it needs a handbag, then it is not a lady kangaroo. handbag → ~lady

converse c) A kangaroo does not need a handbag if it is a lady. ~handbag → lady

4. Terry will study if his brother takes him to the library. library → study

conclusion hypothesis

original a) If his brother takes him to the library, then Terry will study.

inverse b) Because Terry's brother is not taking him to the library, he will not study. ~library → ~study

none c) Terry does not study in the library.

5. Because teachers are fair, students get the grades they deserve. fair → deserve

converse a) Because students get the grades they deserve, their teachers are fair. deserve → fair

contrapositive b) If students don't get the grades they deserve, then their teachers are not fair. ~d → ~f

original c) If teachers are fair, then students get the grades they deserve. fair → deserve

6. $\sim r \to s$

none a) $r \to s$

converse b) $s \to \sim r$

contrapositive c) $\sim s \to r$