

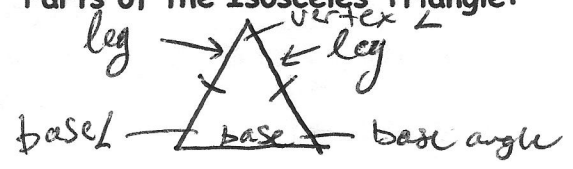
## Section 4.5 Isosceles & Equilateral Triangles

### Geometry Class Notes

**VOCABULARY:**

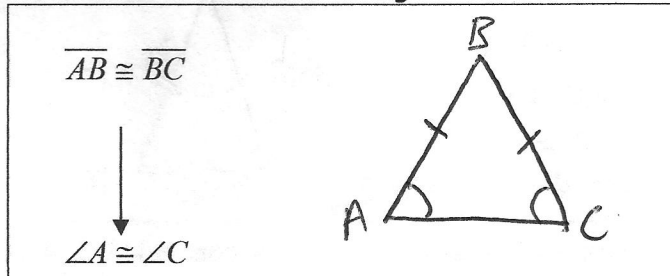
Legs of Isosceles  $\Delta$  :  $\cong$  sides (2)  
 Base of a Isosceles  $\Delta$  : 3rd non-congruent side  
 Vertex  $\angle$  of an Isosceles  $\Delta$  :  $\angle$  formed by the legs  
 Base  $\angle$ 's of an Isosceles  $\Delta$  : LS opposite the legs

**Parts of the Isosceles Triangle:**

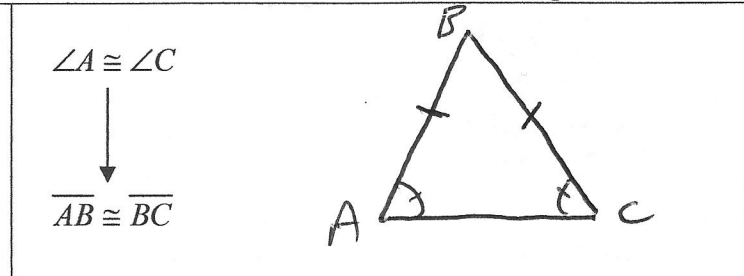


An isosceles  $\Delta$  has symmetry about a line through its vertex angle.

**Theorem 4-3 Isosceles Triangle Theorem**



**Theorem 4-4 Converse of Isosceles Triangle Theorem**

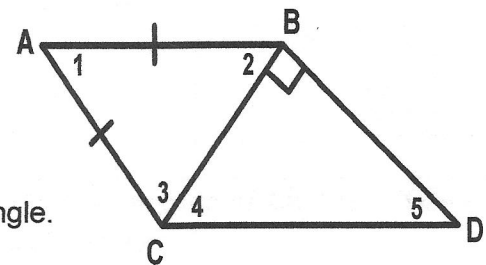


If legs of a triangle are  $\cong$ , then the angles opposite those sides are  $\cong$ .

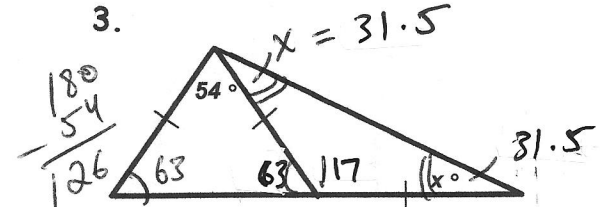
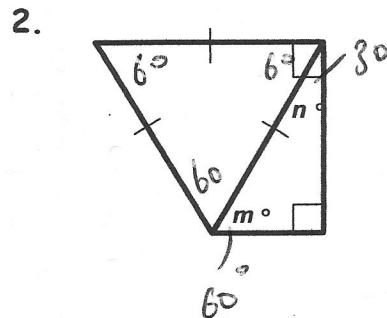
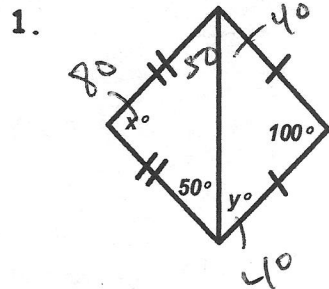
If angles of a triangle are  $\cong$ , then the sides opposite those angles are  $\cong$ .

**EXAMPLES:**

- $\overline{AB}$ ,  $\overline{AC}$  a) Name the legs of the isosceles triangle.  
 $\overline{CD}$  b) Name the hypotenuse of the right triangle.  
 $\angle ABC$ ,  $\angle ACB$  c) Name the base angles of the isosceles triangle.



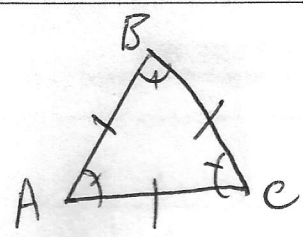
**EXAMPLES: Find the measures for the indicated variables below**



Corollary to Theorem 4-3

If  $\triangle ABC$  is equilateral:

$AB \cong BC \cong AC$



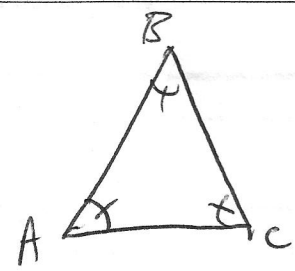
Then:

$\triangle ABC$  is equiangular

Corollary to Theorem 4-4

If  $\triangle ABC$  is equiangular:

$m\angle A = 60^\circ$   
 $m\angle B = 60^\circ$   
 $m\angle C = 60^\circ$

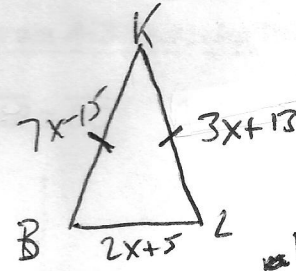


Then:

$\triangle ABC$  is Equilateral

EXAMPLES: Find  $x$  and the measure of each side of the triangles.

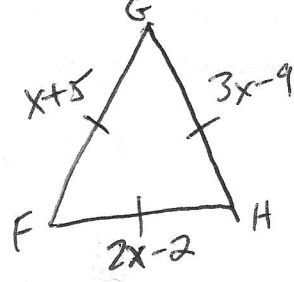
4.  $\triangle BLK$  is an isosceles triangle with a vertex angle being  $\angle K$ . If  $BL = 2x + 5$ ,  $LK = 3x + 13$ , and  $BK = 7x - 15$ , find the length of the base.



$3x + 13 = 7x - 15$   
 $13 = 4x - 15$   
 $4x = 28$   
 $x = 7$

$BL = 2(7) + 5 = 19$

5.  $\triangle FGH$  is equilateral with  $FG = x + 5$ ,  $GH = 3x - 9$ , and  $FH = 2x - 2$ . Find each side.



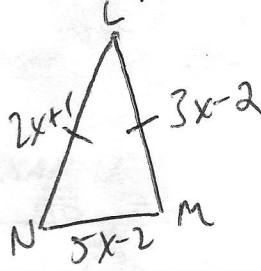
$3x - 9 = 2x - 2$

$x - 9 = -2$

$x = 7$

all sides are 12

6.  $\triangle LMN$  is isosceles,  $\angle L$  is the vertex angle,  $LM = 3x - 2$ ,  $LN = 2x + 1$ , and  $MN = 5x - 2$ .



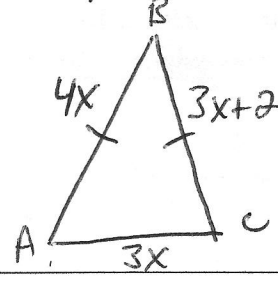
$2x + 1 = 3x - 2$

$1 = x - 2$

$x = 3$

$LN = 7, LM = 7, MN = 13$

7.  $\triangle ABC$  is an isosceles with  $AB = BC$  if  $AB = 4x$ ,  $BC = 3x + 2$ , and  $AC = 3x$ .



$4x = 3x + 2$

$x = 2$

$AB, BC = 8$   
 $AC = 6$

Classify the following triangles.

8. Find the measure of each side of  $\triangle ABC$  with vertices  $A(-1, 5)$ ,  $B(6, 1)$ , and  $C(2, -6)$ .

$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{49 + 16}$

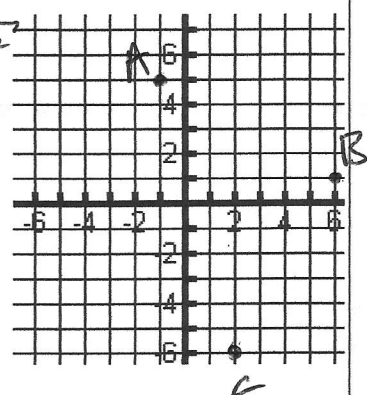
$AB = \sqrt{65}$

$AC = \sqrt{9 + 121}$

$AC = \sqrt{130}$

$BC = \sqrt{16 + 49}$

$BC = \sqrt{65}$



- Isosceles  
 - Right  
 (AB  $\perp$  BC)

9. In  $\triangle NGL$  with a perimeter of 68,  $NG = 3x + 8$ ,  $GL = 6x - 10$ , and  $LN = 2x - 7$ .

$3x + 8 + 6x - 10 + 2x - 7 = 68$

$11x - 9 = 68$

$11x = 77$

$x = 7$

$NG = 29$

$GL = 32$

$LN = 7$

$NG = 29$   
 $GL = 32$   
 $LN = 7$