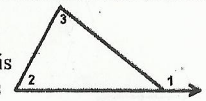
Section 5-5: Inequalities in Triangles
Class Notes

3rd

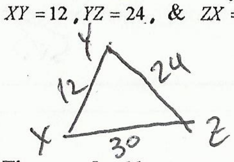
Corollary to Triangle Exterior Angle Theorem

Exterior Angle Inequality Corollary	If an angle is an exterior angle of a triangle, then its measure is greater than either of its remote interior angles.  $m\angle 1 > m\angle 2$ **and** $m\angle 1 > m\angle 3$

Theorem 5 – 10

In a Δ, if one side is longer than another side, then the ∠ opposite the longer side is greater than the ∠ opposite the shorter side. (The biggest ∠ is opposite the biggest side)

Example: List the angles in order from smallest to largest. ΔXYZ has sides of: $XY = 12$, $YZ = 24$, & $ZX = 30$.



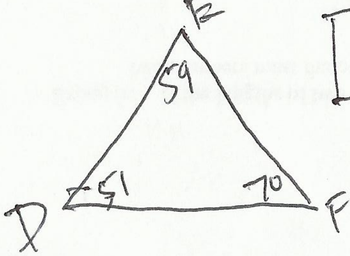
$\boxed{\angle Z, \angle X, \angle Y}$

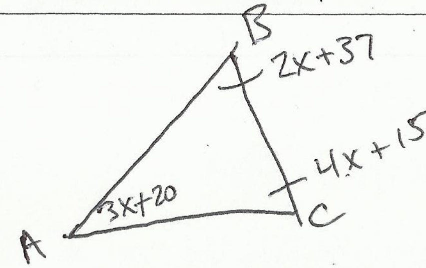
Theorem 5 – 11

In a Δ, if one ∠ is greater than another ∠, then the side opposite the greater ∠ is longer than the side opposite the smaller ∠. (The biggest side is opposite the biggest ∠)

Example: List the sides in order from largest to smallest. ΔDEF has angles of: $m\angle D = 51$, $m\angle E = 59$, & $\angle F = 70$.

$\boxed{\overline{DE}, \overline{DF}, \overline{EF}}$

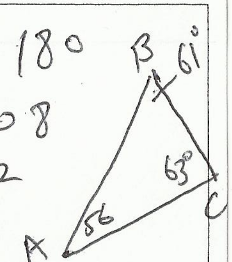




$9x + 72 = 180$

$9x = 108$

$x = 12$



Example: Put the sides in order from the greatest to least in ΔABC if $m\angle A = 3x + 20$, $m\angle B = 2x + 37$, $m\angle C = 4x + 15$.

$\boxed{\overline{AB}, \overline{AC}, \overline{BC}}$

Theorem 5 – 12

Triangle Inequality Theorem	The sum of the lengths of any two sides of a triangle is greater than the length of the third side. **a + b > c**

Example: Which three line segments can be used to form a triangle?

A) 2, 3, 5   B) 5, 10, 18   C) 45, 21, 52

A) $2+3=5$ must be greater — NO

B) $5+10<18$ — NO

C) $45+21>52$, $45+52>21$, $21+52>45$ — YES

Example: If Mrs. Bailey gave Jane four pieces of copper tubing measuring 6m, 7m, 9m and 16m. How many different triangles could Jane make?

6, 7, 9

~~6, 7, 16~~

~~7, 9, 16~~

~~6, 9, 16~~

$\boxed{\text{1 possible } \Delta}$

Example: The lengths of two sides of a triangle are 8 and 13. What are the possible lengths of the third side?

x is shortest

~~$x+8>13$~~ same thing

$8+x>13$

$\boxed{x>5}$

x is longest

$8+13>x$

$\boxed{21>x}$

$\boxed{5<x<21}$

Difference < x < Sum

13−8   13+8

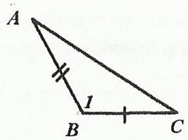
Example: If the lengths of two sides of a triangle are 10 cm and 15cm, between what two numbers must the measure of the third side fall?

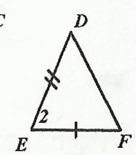
$\boxed{5 < x < 25}$

When you open doors, the width of the door and the width of the doorjamb remain the same, but the angle changes. This causes the distance from the knob to the frame to change. From this, we get: "THE HINGE THEOREM"

SAS Inequality Theorem Hinge Theorem	If 2 sides of a Δ are ≅ to 2 sides of another Δ, then the Δ with the <u>larger</u> included angle also has the <u>longer</u> 3rd side.
SAS ~~SSS~~ Inequality Theorem	If 2 sides of a Δ are ≅ to 2 sides of another Δ, then the Δ with the <u>longest</u> 3rd side also has the <u>largest</u> angle opposite the 3rd side.

Example: $m\angle 1 > m\angle 2$. Compare AC and DF.





1. Can the Hinge Theorem be applied?

YES

2. Which ∠'s are included?

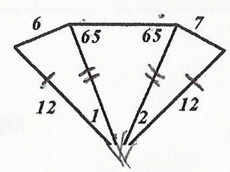
∠1 and ∠2

3. How do AC and DF compare?

AC > DF

Examples

1. Compare $\angle 1$ and $\angle 2$

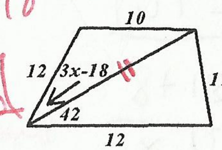


$\boxed{\angle 2 > \angle 1}$

because

7 > 6

2. Find the values of '*x*'



$3x - 18 < 42$

$3x < 60$

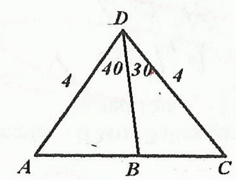
$x < 20$

$3(6) - 18 = 0$

$\boxed{6 < x < 20}$

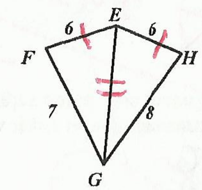
$x > 6$ and $x < 20$

3. Compare AB and BC



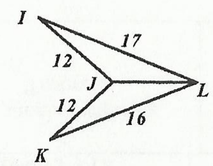
AB > BC

4. Compare $m\angle FEG$ and $m\angle HEG$



$m\angle FEG < m\angle HEG$

5. Compare $m\angle IJL$ and $m\angle KJL$



6. Compare MQ and NP