1st

$\frac{x}{a} = \frac{b}{x}$

Geometric Mean and Similarity in Right Triangles

The geometric mean of two positive numbers a and b is a number x such that $\frac{a}{x} = \frac{x}{b}$.

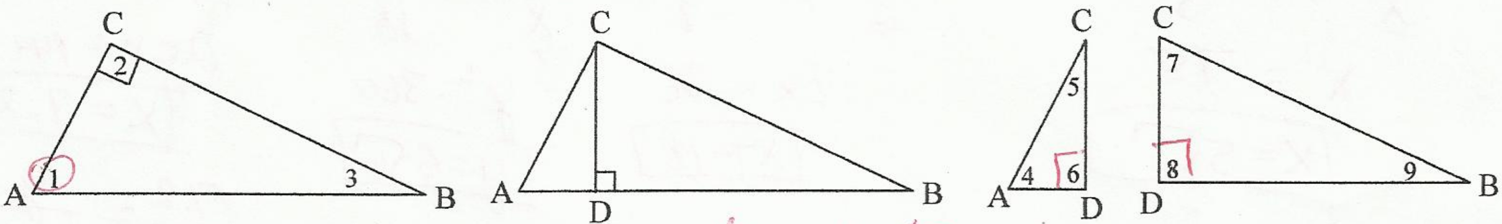
$x = \sqrt{ab}$

Example 1: Find the geometric mean of 4 and 18. $\frac{4}{x} = \frac{x}{18}$ $\boxed{x = 6\sqrt{2}}$

Example 2: Find the geometric mean of $7\sqrt{2}$ and $5\sqrt{6}$. $\frac{7\sqrt{2}}{x} = \frac{x}{5\sqrt{6}}$ $x^2 = 35\sqrt{12} = 70\sqrt{3}$ $x = \sqrt{70\sqrt{3}} \approx \boxed{11}$

Example 3: 6 is the geometric mean between 4 and what value? $\frac{4}{6} = \frac{6}{x}$ $\boxed{x = 9}$

Draw the altitude CD of the right triangle below to create two additional right triangles.



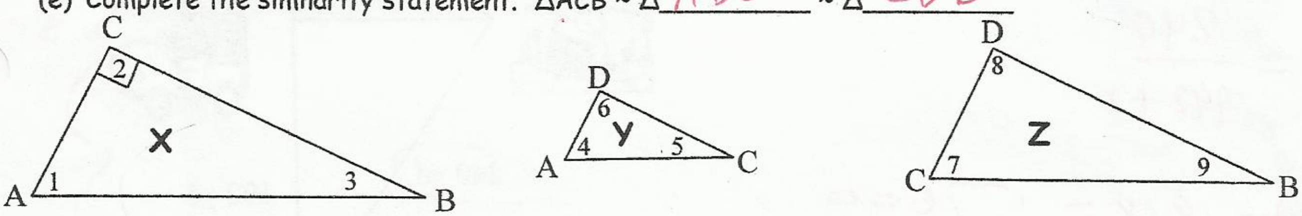
(a) Which angles have the same measure as $\angle 1$? $\angle 4$ and $\angle 7$

(b) Which angles have the same measure as $\angle 2$? $\angle 6$ and $\angle 8$

(c) Which angles have the same measure as $\angle 3$? $\angle 5$ and $\angle 9$

(d) Based on the results, what is true about the three triangles? ~

(e) Complete the similarity statement. $\Delta ACB \sim \Delta$ ADC $\sim \Delta$ CDB



Because the triangles are similar, the corresponding sides are proportional. Here are three of those proportions:

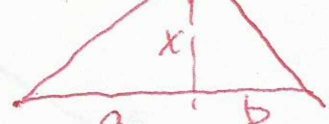
Between X and Y: $\frac{AC}{AD} = \frac{AB}{AC}$, between Y and Z: $\frac{AD}{CD} = \frac{CD}{BD}$, between X and Z: $\frac{BC}{BD} = \frac{AB}{BC}$

Theorem 7-3

The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are ~ to the original triangle and ~ to each other.

Corollary 1 to Theorem 7-3

The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.

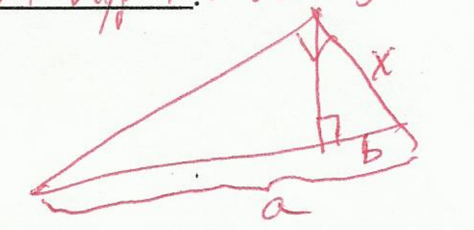


$\frac{a}{x} = \frac{x}{b}$

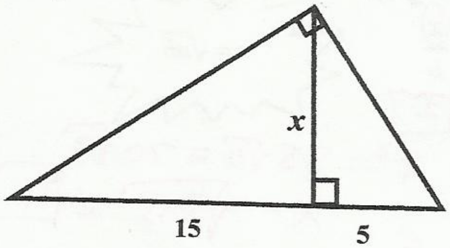
Corollary 2 to Theorem 7-3

The altitude to the hypotenuse of a right triangle separates the hypotenuse so that the length of each leg of the triangle is the geometric mean of the length of the hypotenuse and the length of the adjacent hypotenuse segment.

$\frac{a}{x} = \frac{x}{b}$



Example 4: Solve for x.

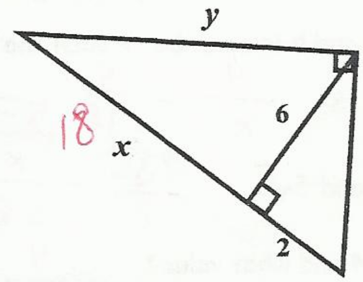


$$\frac{15}{x} = \frac{x}{5}$$

$$x^2 = 75$$

$$\boxed{x = 5\sqrt{3}}$$

Example 5: Solve for x and y.

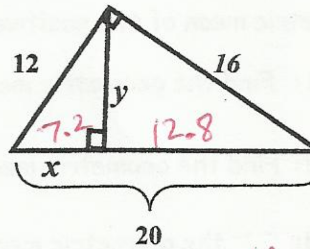


$$\frac{x}{6} = \frac{6}{2} \qquad \frac{20}{y} = \frac{y}{18}$$

$$2x = 36 \qquad y^2 = 360$$

$$\boxed{x = 18} \qquad \boxed{y = 6\sqrt{10}}$$

Example 6: Solve for x.



$$\frac{x}{12} = \frac{12}{20}$$

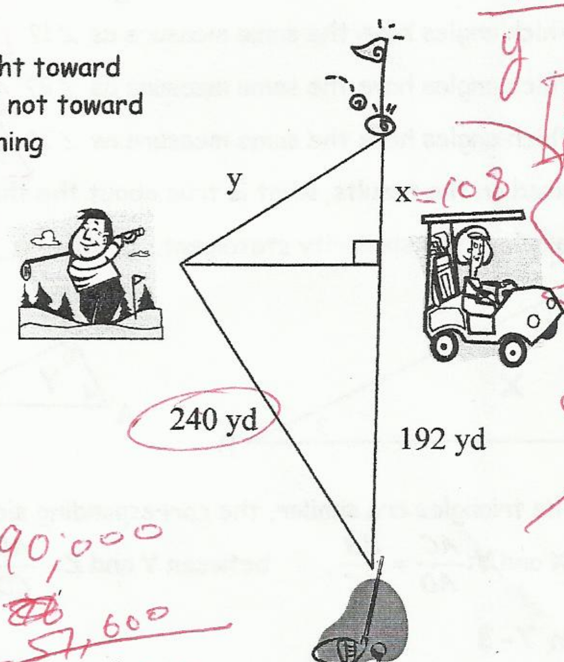
$$20x = 144$$

$$\boxed{x = 7.2}$$

$$\frac{7.2}{y} = \frac{y}{12.8}$$

$$\boxed{y = 9.6}$$

Example 7: At a golf course, Maria drove her ball 192 yd straight toward the cup. Her brother Gabriel drove his ball straight 240 yd, but not toward the cup. The diagram shows the results. Find x and y, the remaining distances from the cup.



3=0

$$\frac{192}{240} = \frac{240}{192 + x}$$

$$36,864 + 192x = 57,600$$

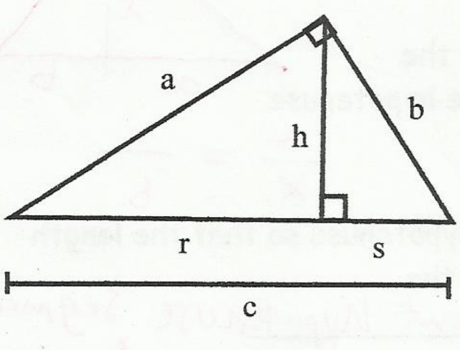
$$192x = 20,736$$

$$\boxed{x = 108}$$

$$90,000$$
$$- 57,600$$
$$32,400$$

$$\boxed{y = 180}$$

Example 8:
Complete each proportion us the triangle below:



i) $\frac{r}{h} = \frac{h}{?}$ s — h is the geometric mean of what?

ii) $\frac{c}{a} = \frac{a}{?}$ r — a is the geometric mean of what?

iii) $\frac{s}{?} = \frac{?}{c}$ b — What is the geometric mean of s and c?